CALCULATION OF THE PARAMETERS OF A TWO-PHASE MIXTURE FLOW IN A CHANNEL OF VARIABLE CROSS SECTION

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The case of steady-state one-dimensional flow in a channel of variable cross section is examined for a two-phase gas-solids mixture. Equations relating the parameters of a gas-dust mixture and the dimensionless gas velocity are derived on the assumption that the velocities and temperatures of the two phases are equal. A formula is obtained for the coefficient of polytropy with allowance for flow friction against the channel walls.

The flow of a gas suspension in a channel of variable cross section was investigated in [1–4]. Expressions were obtained for the flow parameters with allowance for the velocity and temperature lag of the solids. Estimates of the particle velocities and temperatures in [1] and the experimental data of [3] showed that the velocities and temperatures of the two phases almost coincide, if the particle size is not greater than 10 μ. On the basis of these results we will derive approximate formulas for the parameters of a two-phase mixture containing small particles.

We will consider a channel of variable cross section through which flows a two-phase mixture of an ideal gas and small particles (diameter 1–10 μ). In view of the smallness of the particles the medium may be regarded as homogeneous. We assume that the flow is stationary, that there is no heat exchange between the channel walls and the flow, and that the heat released as a result of flow friction against the channel walls is absorbed only by the flow. We write the following equations [5] for the gas velocity and temperature:

$$(M^2-1)\frac{dM^2}{M^2}= 2\left(1+\frac{k-1}{2}M^2\right)\frac{dF}{F}-\frac{k+1}{a^2}dL_s- -\frac{2k}{a^2}\left(1+\frac{k-1}{2}M^2\right)dL_g-\frac{k-1}{a^2}(1+kM^2)\,dQ, \quad (1)$$

$$(M^2-1)\frac{dT}{T}=-(k-1)M^2\frac{dF}{F}+\frac{k-1}{a^2}dL_s+ +\frac{k(k-1)}{a^2}dL_g+\frac{k-1}{a^2}(kM^2-1)\,dQ, \quad (2)$$

where

$$dL_s=\frac{1}{2}\mu\, dW^2, \quad (3)$$

$$dL_g=\frac{1}{2}\zeta\, dW^2, \quad (4)$$

$$dQ=-\mu c_s dT. \quad (5)$$

Here, dL_s is the kinetic energy increment of μ kg of solid phase due to the work of expansion of 1 kg of gas; dL_g is the energy expended by 1 kg of gas phase in overcoming drag forces; dQ is the heat transferred between μ kg of particles and 1 kg of gas.

Considering that $dW^2 = a^2dM^2 + M^2da^2$, we can write (3) and (4) as follows:

$$dL_s=\frac{1}{2}\mu(a^2dM^2+M^2da^2), \quad (3')$$

$$dL_g=\frac{1}{2}\zeta(a^2dM^2+M^2da^2). \quad (4')$$

Solving system (1)–(2) with allowance for (3'), (4'), and (5), we obtain the following expressions for the flow parameters in two sections of the channel:

$$\frac{T_2}{T_1}=A, \quad (6)$$

$$\frac{\rho_2}{\rho_1}=A^{\frac{(1+\mu)\left(1+\frac{k\mu c_s}{c_p}\right)+\zeta k\left(1+\frac{\mu c_s}{c_p}\right)}{(k-1)(1+\mu)}}, \quad (7)$$

$$\frac{p_2}{p_1}=A^{\frac{k(1+\mu)\left(1+\frac{\mu c_s}{c_p}\right)+\zeta k\left(1+\frac{\mu c_s}{c_p}\right)}{(k-1)(1+\mu)}}, \quad (8)$$

where

$$A=\frac{M_1^2(k-1)(1+\mu)+2\left(1+\frac{\mu c_s}{c_p}\right)}{M_2^2(k-1)(1+\mu)+2\left(1+\frac{\mu c_s}{c_p}\right)}.$$

When $\mu = 0$ and $\zeta = 0$ Eqs. (6)–(8) reduce to the familiar formulas of gas dynamics [5].

Substituting (7) and (8) in the polytropic equation

$$\frac{p_2}{p_1}=\left(\frac{\rho_2}{\rho_1}\right)^n,$$

we find the coefficient of polytropy

$$n=\frac{k(1+\mu)\left(1+\frac{\mu c_s}{c_p}\right)+\zeta k\left(1+\frac{\mu c_s}{c_p}\right)}{(1+\mu)\left(1+k\mu\frac{c_s}{c_p}\right)+\zeta k\left(1+\frac{\mu c_s}{c_p}\right)}. \quad (9)$$

If we disregard gas friction against the channel walls, Eq. (9) goes over into [6]

$$n=\left(k+\frac{\mu c_s}{c_v}\right)\left(1+\frac{\mu c_s}{c_v}\right)^{-1}.$$

From (2), using (6), we obtain an expression for the area ratio:

$$\frac{F_2}{F_1} = \frac{M_1}{M_2} A^{-\frac{(1+\mu)\left(1+k+2k\mu\frac{c_s}{c_p}\right)+2\zeta k\left(1+\frac{\mu c_s}{c_p}\right)}{2(k-1)(1+\mu)}} \tag{10}$$

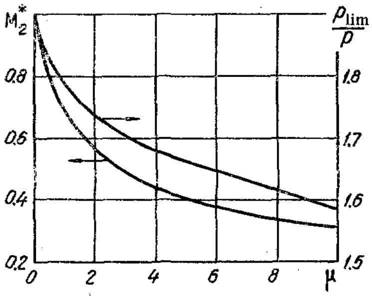
If we use (9), we can reduce Eqs. (7), (8), and (10) to the more compact form

$$\frac{\rho_2}{\rho_1} = A^{\frac{1}{n-1}}, \tag{7'}$$

$$\frac{p_2}{p_1} = A^{\frac{n}{n-1}}, \tag{8'}$$

$$\frac{F_2}{F_1} = \frac{M_1}{M_2} A^{-\frac{1+n}{2(n-1)}}.$$

We will determine the Mach number M_2^* in the channel section having the least diameter. For this purpose



Limiting flow velocity and limiting pressure at channel inlet as functions of flow-rate concentration μ (kg/kg) for a graphite-air mixture.

we take the derivative of (10) with respect to M_2 and equate it to zero. As a result we obtain

$$M_2^* = \left(1+\frac{\mu c_s}{c_p}\right)^{\frac{1}{2}} \times$$

$$\times \left[(1+\mu)\left(1+k\mu\frac{c_T}{c_p}\right)+\zeta k\left(1+\frac{\mu c_s}{c_p}\right)\right]^{-\frac{1}{2}} \tag{11}$$

or in shortened form

$$M_2^* =$$

$$= \left[(n-1)\left(1+\frac{\mu c_s}{c_p}\right)\right]^{\frac{1}{2}} \left[(k-1)(1+\mu)\right]^{-\frac{1}{2}}. \tag{11'}$$

The dependence of M_2^* on the solids concentration for an air flow with graphite particles without allowance for friction is shown in the figure.

We will call the flow velocity corresponding to M_2^* the "limiting" velocity and find the minimum pressure at the channel inlet for which a decrease of pressure in the ambient medium does not casue an increase in flow velocity. Assuming, for simplicity that $M_1 = 0$, from (8') and (11') we obtain

$$\frac{p_{lim}}{p} = \left(\frac{n+1}{2}\right)^{\frac{n}{n-1}}. \tag{12}$$

At $\mu = 0$ and $\zeta = 0$ we have the case of isentropic flow

$$\frac{p_{lim}}{p} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}.$$

The dependence of p_{lim}/p on μ for a graphite-air mixture at $\zeta = 0$ is shown in the figure. It is clear from the graph that p_{lim} decreases with increase in solids concentration. If the operating regime of a convergent nozzle is such that for pure gas the pressure at the nozzle exit is equal to the ambient pressure, then introducing particles into the flow leads to underexpansion. It is easy to show that this also applies to a nozzle with a divergent section (Laval nozzle).

After the experimental determination of the resistance coefficient, the formulas derived can be used to calculate the parameters of a two-phase flow containing particles up to 10 μ in size.

NOTATION

M is the Mach number; a is the speed of sound in the isentropic gas flow; F is the area of the channel cross section; k is the isentropic exponent; T is the temperature of the gas (particles); ζ is the nozzle-resistance coefficient for two-phase flow; μ is the flow-rate concentration; ρ is the gas density; p is the gas pressure; c_p is the specific heat of 1 kg gas at constant pressure; c_v is the specific heat of 1 kg gas at constant volume; c_s is the specific heat of 1 kg solid phase.

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